HEAT TRANSFER UNDER RADIATION EQUILIBRIUM IN A RECTANGULAR CHANNEL

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Local and integral resultant radiation fluxes on the wall of an infinite rectangular channel are determined numerically for the radiation equilibrium case. The error in the plane layer approximation is analyzed.

The working spaces of a number of technological aggregates, e.g., certain kinds of metallurgical furnaces, can be represented in the form of a channel with a rectangular cross section. The approximation of an infinite plane layer [1] is often used in computations of the radiation heat transfer in these apparatus. A quantitative estimation of the validity of such an approximation has not yet been performed, however, it is important for practice.

Let us examine the radiation heat transfer between diffusely grey walls of a rectangular channel of infinite extent. The channel is filled with a radiating, absorbing, anisotropically dissipating grey medium. The medium is in a state of local radiation equilibrium. The wall temperatures are given — the local densities of the resultant radiation fluxes on the walls. We shall solve the problem by using the radiation integral equations [2]. A grey medium in radiation equilibrium can be formally considered purely dissipating (γ = 1,0) since each volume element completely reradiates all the incident radiation. The real scattering index should be replaced by the effective value taking account of the intrinsic radiation of the medium [3]. In such an approach the temperature distribution in the medium need not be known to find the resultant radiation flux density on the wall. We shall take account of radiation scattering in the quasi-one-dimensional approximation [4], i.e., we approximate the index by an ultimately extended "front-to-back" in the ray direction. In this case the need for an integral equation for the volume radiation density drops out and the problem reduces to an integral equation for the surface radiation density

$$q_e(N) = q_i(N) + r(N)\left[\int\limits_F q_e(N')\,K(N,\,N')\,dF_{N'} + q_e(N)\int\limits_F K^*(N,\,N')\,dF_{N'}\right]. \tag{1}$$

which differs from the analogous equation presented in [2] in the second term in the square brackets, denoting the radiation flux in [1] that is incident on the surface element dF_N under consideration because of reflection of the effective radiation of this surface from the dissipating medium. We obtained the following expressions for the kernels of (1):

$$K = \frac{h^2}{\pi l^3}\int\limits_{-1}^{+1} D\sqrt{1-\mu^2}\,d\mu, \tag{2}$$

$$K^* = \frac{h^2}{2l^3} - K. \tag{3}$$

In the quasi-one-dimensional approximation, the ray transmission function at γ = 1 has the form

$$D = \frac{2}{2 + \dfrac{(1-\bar{\mu})\,kl}{\sqrt{1-\mu^2}}}. \tag{4}$$

Substituting (4) into (2), we obtain

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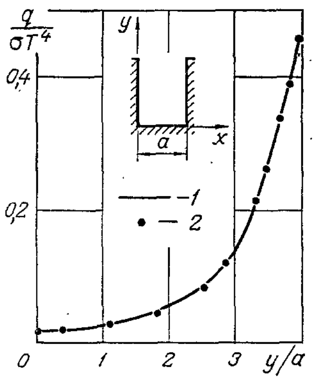

Fig. 1

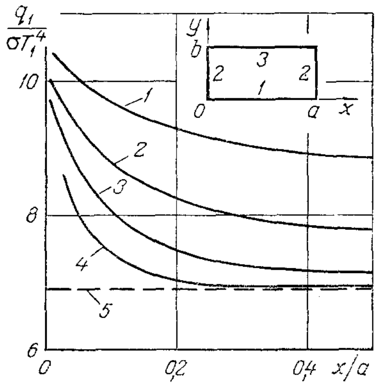

Fig. 2

Fig. 1. Comparison of results of computing the dimensionless resultant radiation flux density: 1) data from [5]; 2) authors' computations.

Fig. 2. Dimensionless resultant radiation flux densities on the lower base of the channel for $\tau = 1.12$; $T_2/T_1 = T_3/T_1 = 2$; $r_1 = 0.02$; $r_2 = r_3 = 0.2$; 1) $a/b = 1$; 2) 2.0; 3) 4.0; 4) 8.0; 5) $a/b \to \infty$.

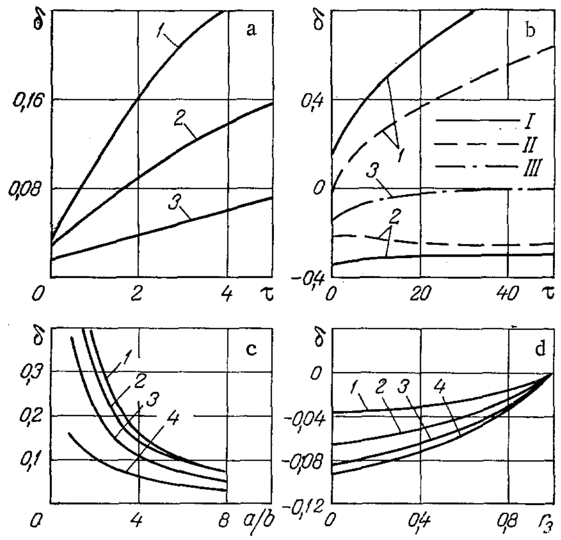

Fig. 3. Influence of τ, a/b, T_2/T_1, r on the magnitude of the error in the approximation of a plane layer δ: a) $T_2/T_1 = T_3/T_1$, $r_1 = 0.8$, $r_2 = r_3 = 0.2$ 1) $a/b = 1.0$, 2) 2.0, 3) 5.0 b) $T_3/T_1 = 2.0$ $a/b = 2.0$, $r_1 = 0.02$, $r_2 = r_3 = 0.2$, 1) $T_2/T_1 = 2.0$, 2) 1.0, 3) adiabatic side walls (I—$r_2 = 0.2$; II—0.8; III—1.0); c) $T_2/T_1 = T_3/T_1$, $r_1 = 0.02$, $r_2 = r_3 = 0.2$, 1) $\tau = 4.8$; 2) 2.8; 3) 1.12; 4) 0.0; d) $T_3/T_1 = 2.0$; $a/b = 2.0$; $r_2 = 1.0$; 1) $r_1 = 0.8$; 2) 0.5; 3) 0.2; 4) 0.001; $\tau = 2.0$.

$$K = \frac{h^2}{\pi l^3} \int_{-1}^{+1} \frac{(1-\mu^2)\, d\mu}{\sqrt{1-\mu^2} + pl}, \quad (5)$$

where

$$h = \begin{cases} y_N - y_{N'}, & \text{for } y_N = 0 \text{ or } y_N = b. \\ x_N - x_{N'}, & \text{for } 0 < y_N < b; \end{cases}$$

$$l = \sqrt{(x_N - x_{N'})^2 + (y_N - y_{N'})^2}; \quad p = \frac{k}{2}(1 - \bar{\mu});$$

$x_{N'}$, y_N, $y_{N'}$ $y_{N'}$ are the coordinates of the considered and running points on the wall surface in the channel cross section.

The integral equation (1) was solved numerically by the method of algebraic approximation [2]. The number of partitions of the boundary surface in the channel cross section was $n \sim 100$, which assured $\leqslant 1\%$ error in the determination of the radiation flux density. The integration in (5) was by the Gauss quadrature method. The integral in (1) were approximated algebraically by the rectangle formula. To improve the accuracy of the solution, the partition intervals were diminished in a geometric progression upon approaching the corners of the cross section. The system of linear algebraic equations obtained ($n \sim 100$) for the effective radiation flux density was solved by the Gauss method. Values of the resultant flux densities were determined from the known dependence

$$q = \frac{1}{r}[(1-r)q_e - q_i]. \tag{6}$$

The accuracy of the numerical solution was estimated by comparing with the data in [5], where the problem is solved for an isothermal cavity filled with a transparent medium. Satisfactory agreement between the results is obtained (Fig. 1).

Computations by the method elucidated were performed in a broad range of parameter values that can be encountered in practice (Figs. 2 and 3). As an illustration, let us examine the case of external heat transfer in the charge heating chamber of a two-bath steel-melting furnace, to which the following values of the parameters correspond approximately: $a/b = 2$, $T_2/T_1 = T_3/T_1 = 2$, $r_1 = 0.02$, $r_2 = r_3 = 0.2$, $k = 0.67\ m^{-1}$, $b = 4$ m, $\bar{\mu} = 0.58$. The subscript 1 refers to the lower base of the rectangular section (charge), 2 to the side walls, and 3 to the upper base (crown). The theoretical solution of this problem has been obtained in [1] in the plane layer approximation, and the results of an experimental study are presented in [6]. It follows from a comparison of [1] and [6] that the medium at the exit from the chamber is in an almost radiation equilibrium state. Lowering the values of the thermal flux density in [1] can explain neglecting the influence of the chamber side walls. Local resultant radiation flux densities on the lower base of the channel are shown in Fig. 2 for different ratios between the channel width a and its height b. The values of $q_1/\sigma T_1^4$ near the side walls differ noticeably from the values at the center of the channel. As the ratio a/b grows, the influence of the side walls attenuates and the magnitudes of the radiation flux densities tend to the limit value that is valid for a plane layer ($a/b \to \infty$). The error in determining the resultant fluxes in modeling an infinite rectangular channel by a plane layer is denoted by

$$\delta = \frac{Q - Q_0}{Q}, \tag{7}$$

where $Q = \int_0^a q_1 dx$ is the resultant radiation flux on the lower base, Q_0 is the same in the plane layer approximation [7]. The dependence of the quantity $\tau = kb(1-\bar{\mu})$ the different parameters is presented in Fig. 3, where the influence of the parameter a/b, T_2/T_1, characterizing the optical properties of the medium, is also shown. The computation is performed for different ratios a/b, T_2/T_1 and different values of the side wall reflexivity r_2. It is obtained that the absolute value of δ diminishes as r_2 grows, and reaches minimal values for adiabatic side walls ($r_2 = 1$). If the side wall temperature equals the temperature of the upper base, then δ is independent of T_2/T_1 since the local radiation flux density is linear in the difference in fourth powers of the temperature of the lower base and of the surrounding walls. Otherwise, δ is a function of T_2/T_1. Thus, for $T_3/T_1 = 2$ and a change in T_2/T_1 from 1 to 2, the quantity δ changes sign from minus to plus (Fig. 3b). For "cold" ($T_2/T_1 = 1$) side walls, δ is practically independent of τ, while for "hot" ($T_2/T_1 = 2$) walls a growth of δ is observed with the increase in τ. The dependence of δ on a/b is shown in Fig. 3c for different τ. It follows therefrom that in computing the heat transfer in the charge heating chamber in the plane layer approximation, the resultant radiation flux on the charge is reduced. The influence of negative reflexivities of the upper and lower bases on the quantity δ is shown in Fig. 3d for adiabatic side walls. The maximum error is observed for absolutely black surfaces of the bases.

The method proposed and the computation results are recommended for utilization to estimate the error in the plane layer approximation when modeling radiation heat transfer in rectangular chambers.

NOTATION

T, absolute temperature; q, radiation flux density; Q, resultant radiation flux; σ, Stefan–Boltzmann constant; r, surface reflexivity; k, linear attenuation factor; γ, ratio of scattering to attenuation coefficients; $\bar{\mu}$, effective mean cosine of the scattering angle in an elementary scattering act; D, transmission function; l, ray pathlength; a, channel width; b, channel height; x, y, coordinates. Subscripts: e, effective radiation; c, intrinsic radiation; 1, lower base of channel cross section; 2, side walls; 3, upper base.

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PROBLEM OF THE NONSTATIONARY STATE OF HEAT- AND MASS-TRANSFER PROCESSES IN BINARY GASEOUS MIXTURES

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Nonstationary heat- and mass-transfer processes in gaseous mixtures are considered, and expressions are obtained for the heat-diffusion ratio and for the contribution of diffusion thermal conduction in conductive heat transfer.

Molecular heat- and mass-transfer processes in gaseous mixtures are characterized by effective values of the thermal conductivity and thermal diffusion ratio, and these two (effective) characteristics (transfer processes) are mutually related and may differ in value in the stationary and nonstationary states.

Despite the large number of papers published on the subject, the mechanism of the phenomenon of thermal diffusion in gaseous mixtures is still unclear even in the case of mixtures of monotonic gases. Experimental methods of determining the thermal diffusion constant of gaseous mixtures are usually stationary, since at the present time there is not even a theory which describes the nonstationary state of thermal diffusion.

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